

Majorana neutrinos and non minimal lepton mass textures

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Abstract

In the light of the recent measurement of the leptonic mixing angle θ_{13} , implications of the latest mixing data have been investigated for non-minimal textures of lepton mass matrices assuming the neutrinos to be Majorana like. Large number of possible texture specific lepton mass matrices have been examined for their compatibility with the lepton mixing data in the case of normal hierarchy, inverted hierarchy and degenerate scenario of neutrino masses. Specifically, apart from other phenomenological quantities the implications of the lepton mixing angle θ_{13} have been investigated on the lightest neutrino mass as well as the effective Majorana mass $\langle m_{ee} \rangle$.

1 Introduction

The last few years have witnessed spectacular advances in fixing the neutrino masses and mixing parameters through various solar [1]-[7], atmospheric [8], reactor [9],[10] and accelerator [11]-[13] neutrino experiments. In this context, the recent observations [14], [15] regarding the unexpectedly ‘large’ value of the mixing angle θ_{13} have led to an increased interest in investigating the implications of the neutrino oscillations phenomenology. In particular, the nonzero value of θ_{13} could lead to the possibility of existence of CP violation in the leptonic sector, therefore leading to an increased amount of activity in this area. These observations have also deepened

the mystery of flavor mixings as the patterns of quark and lepton mixing angles now look to be significantly different. Noting that the mixing angles and CP violating phases are very much related to the corresponding mass matrices, one essentially has to formulate the fermion mass matrices to unravel some of the deeper aspects of flavor physics. In case one assumes unification or quark-lepton complementarity [16], it becomes desirable to understand the quark and lepton mixings from the same perspective as far as possible, thus making the formulation of viable fermion mass matrices all the more complicated.

In the absence of any compelling theory of flavor dynamics, one usually resorts to phenomenological models. In this context, the concept of texture specific mass matrices, introduced implicitly by Weinberg [17] and explicitly by Fritzsch [18], has got a good deal of attention in the literature, for details we refer the reader to a recent review article [19].

Considering neutrinos to be Majorana particles, after the recent measurement of θ_{13} , a few analyses have been carried out for texture specific mass matrices in the non-flavor basis. In particular, Fukugita *et al.* [20] have investigated the implications of angle θ_{13} on minimal texture mass matrices (Fritzsch-like texture six zero) for normal hierarchy of neutrino masses. This analysis has been extended further by Fakay *et al.* [21] wherein for all hierarchies of neutrino masses, Fritzsch-like texture zero and five zero mass matrices have been examined in detail. However, a detailed and comprehensive analysis for non-Fritzsch like lepton mass matrices with five texture zeroes and beyond is yet to be carried out. In this context, it may be noted that Branco *et al.* [22] have carried out a detailed analysis for all possible structures of texture four zero lepton mass matrices, however similar attempts have not been carried out after the recent measurements of θ_{13} .

In the present paper, we have attempted to carry out detailed calculations pertaining to lepton mass matrices with non-minimal textures for all the three possibilities of neutrino masses. In particular, the analysis has been carried out for Fritzsch-like texture two zero mass matrices as well as for all possibilities of texture four zero and five zero lepton mass matrices. The compatibility of these texture

specific mass matrices has been examined by plotting the parameter space corresponding to any two mixing angles. Further, the implications of mixing angles on the lightest neutrino mass as well as the effective Majorana mass $\langle m_{ee} \rangle$ have also been investigated.

The detailed plan of the paper is as follows. In Section (2), we detail the essentials of the formalism regarding the texture specific mass matrices. Inputs used in the present analysis have been given in Section (3) and the discussion of the calculations and results have been presented in Section (4). Finally, Section (5) summarizes our conclusions.

2 Texture specific mass lepton mass matrices in the Standard Model and the PMNS matrix

Using the facility of weak basis (WB) transformations [24], it can be shown that the most general lepton mass matrices within the framework of standard model (SM) can be expressed as

$$M_l = \begin{pmatrix} C_l & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix} C_\nu & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}, \quad (1)$$

M_l and $M_{\nu D}$ corresponding to charged lepton and Dirac-like neutrino mass matrices respectively. Both the matrices are texture 1 zero type with $A_{l(\nu)} = |A_{l(\nu)}|e^{i\alpha_{l(\nu)}}$ and $B_{l(\nu)} = |B_{l(\nu)}|e^{i\beta_{l(\nu)}}$. Further, to facilitate the formulation of phenomenological mass matrices, which perhaps are compatible with the GUT scale mass matrices, it has been suggested [26] that in order to avoid fine tuning amongst the elements of the mass matrices, these should follow a ‘natural hierarchy’ i.e. $(1, 1), (1, 2), (1, 3) \lesssim (2, 2), (2, 3) \lesssim (3, 3)$.

For the complete diagonalizing matrix pertaining to structure presented in equation (1) we refer the reader to [23], however to understand the relationship between

diagonalizing transformations for different hierarchies of neutrino masses and for the charged lepton case as detailed in [21], we present here the first element of the diagonalizing transformation O_k ,

$$O_k(11) = \sqrt{\frac{(E_k - m_1)(D_k + E_k - m_1 - m_2)(D_k + E_k - m_1 - m_3)}{(D_k + 2E_k - m_1 - m_2 - m_3)(m_1 - m_2)(m_1 - m_3)}}, \quad (2)$$

$m_1, -m_2, m_3$ being the eigen values of a general mass matrix M_k with structure as given in equation (1). In the case of charged leptons, because of the hierarchy $m_e \ll m_\mu \ll m_\tau$, the mass eigenstates can be approximated to the respective flavor eigenstates, i.e. $m_{l1} \simeq m_e$, $m_{l2} \simeq m_\mu$ and $m_{l3} \simeq m_\tau$, and thus one can obtain the first element of the matrix O_l from the equation (2), by replacing m_1, m_2, m_3 by m_e, m_μ, m_τ , e.g.,

$$O_l(11) = \sqrt{\frac{(E_l - m_e)(D_l + E_l - m_e - m_\mu)(D_l + E_l - m_e - m_\tau)}{(D_l + 2E_l - m_e - m_\mu - m_\tau)(m_e - m_\mu)(m_e - m_\tau)}} \quad (3)$$

Equation (2) can also be used to obtain the first element of diagonalizing transformation for Majorana neutrinos, assuming normal hierarchy, defined as $m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}$, and also valid for the degenerate case defined as $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3}$, by replacing m_1, m_2, m_3 by $\sqrt{m_{\nu_1} m_R}, \sqrt{m_{\nu_2} m_R}, \sqrt{m_{\nu_3} m_R}$, e.g.,

$$O_\nu(11) = \sqrt{\frac{(E_\nu - \sqrt{m_{\nu_1}})(D_\nu + E_\nu - \sqrt{m_{\nu_1}} - \sqrt{m_{\nu_2}})(D_\nu + E_\nu - \sqrt{m_{\nu_1}} - \sqrt{m_{\nu_3}})}{(D_\nu + 2E_\nu - \sqrt{m_{\nu_1}} - \sqrt{m_{\nu_2}} - \sqrt{m_{\nu_3}})(\sqrt{m_{\nu_1}} - \sqrt{m_{\nu_2}})(\sqrt{m_{\nu_1}} - \sqrt{m_{\nu_3}})}}, \quad (4)$$

where m_{ν_1}, m_{ν_2} and m_{ν_3} are neutrino masses. In the same manner, one can obtain the elements of diagonalizing transformation for the inverted hierarchy case, defined as $m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2}$, by replacing m_1, m_2, m_3 in equation (2) with $\sqrt{m_{\nu_1} m_R}, -\sqrt{m_{\nu_2} m_R}, -\sqrt{m_{\nu_3} m_R}$, e.g.,

$$O_\nu(11) = \sqrt{\frac{(E_\nu - \sqrt{m_{\nu_1}})(D_\nu + E_\nu - \sqrt{m_{\nu_1}} - \sqrt{m_{\nu_2}})(D_\nu + E_\nu - \sqrt{m_{\nu_1}} + \sqrt{m_{\nu_3}})}{(D_\nu + 2E_\nu - \sqrt{m_{\nu_1}} - \sqrt{m_{\nu_2}} + \sqrt{m_{\nu_3}})(\sqrt{m_{\nu_1}} - \sqrt{m_{\nu_2}})(\sqrt{m_{\nu_1}} + \sqrt{m_{\nu_3}})}}. \quad (5)$$

The other elements of diagonalizing transformations in the case of neutrinos as well

as charged leptons can similarly be found. It can be shown that using the seesaw mechanism, the effective neutrino mass matrix can be expressed as,

$$M_\nu = P_{\nu D} O_{\nu D} \frac{(M_{\nu D}^{diag})^2}{(m_R)} O_{\nu D}^T P_{\nu D}, \quad (6)$$

m_R being the right handed neutrino mass scale. Further, the lepton mixing matrix can be expressed as

$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D}, \quad (7)$$

where $Q_l P_{\nu D}$, without loss of generality, can be taken as $(e^{i\phi_1}, 1, e^{i\phi_2})$, ϕ_1 and ϕ_2 being related to the phases of mass matrices and can be treated as free parameters.

3 Inputs used for the analysis

In the present analysis, we have made use of the results of a latest global three neutrino oscillation analysis [27], in table (1) we present the 1σ and 3σ ranges of the neutrino oscillation parameters.

Parameter	1σ range	3σ range
$\Delta m_{sol}^2 [10^{-5} eV^2]$	(7.32-7.80)	(6.99-8.18)
$\Delta m_{atm}^2 [10^{-3} eV^2]$	(2.33-2.49)(NH); (2.31-2.49) (IH)	(2.19-2.62)(NH); (2.17-2.61)(IH)
$\sin^2 \theta_{13} [10^{-2}]$	(2.16-2.66)(NH); (2.19-2.67)(IH)	(1.69-3.13)(NH); (1.71-3.15) (IH)
$\sin^2 \theta_{12} [10^{-1}]$	(2.91-3.25)	(2.59-3.59)
$\sin^2 \theta_{23} [10^{-1}]$	(3.65-4.10)(NH); (3.70-4.31)(IH)	(3.31-6.37)(NH); (3.35-6.63)(IH)

Table 1: The 1σ and 3σ ranges of neutrino oscillation parameters presented in [27].

While carrying out the analysis, the lightest neutrino mass, m_1 for the case of NH and m_3 for the case of IH, is considered as a free parameter, which is explored within the range $10^{-8} \text{ eV} - 10^{-1} \text{ eV}$ for all the three possible mass hierarchies of neutrinos i.e. normal, inverted and degenerate scenario. In the absence of any constraint on the phases, ϕ_1 and ϕ_2 have been given full variation from 0 to 2π . The parameters $D_{l,\nu}$ and $C_{l,\nu}$ have been considered as free parameters, however, they have been constrained such that diagonalizing transformations O_l and O_ν always remain real.

4 Results and discussions

4.1 Texture two zero lepton mass matrices

To examine the compatibility of texture two zero lepton mass matrices given in equation (1) with the recent mixing data, we carry out a detailed analysis pertaining to all three possible neutrino mass hierarchies. Firstly, we attempt to examine the compatibility of these texture two zero matrices with the inverted hierarchy of neutrino masses. To this end, in figure (1) we present the plot showing the parameter space corresponding to the mixing angle s_{12} along with s_{13} . Giving full allowed variation to other parameters, figure (1) has been obtained by constraining the angle s_{23} by its 3σ experimental bound. The blank rectangular regions in this plot shows the experimentally allowed 3σ regions for s_{12} and s_{13} . Interestingly, a general look at this plot reveals the viability of inverted hierarchy of neutrino masses for the texture two zero mass matrices presented in eqn.(1) as can be seen from the significant overlap between the parameter space allowed by this structure with the experimentally allowed 3σ region.

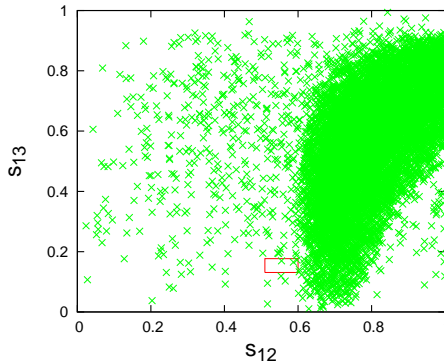


Figure 1: Plot showing the parameter space corresponding to s_{12} and s_{13} for texture two zero mass matrices (inverted hierarchy).

After examining the viability of inverted hierarchy of neutrino masses for the mass matrix structure given in equation (1), we now examine the compatibility of these matrices for the normal hierarchy case. To this end, in figure (2) we present the plots showing the parameter space allowed for two mixing angles when the third

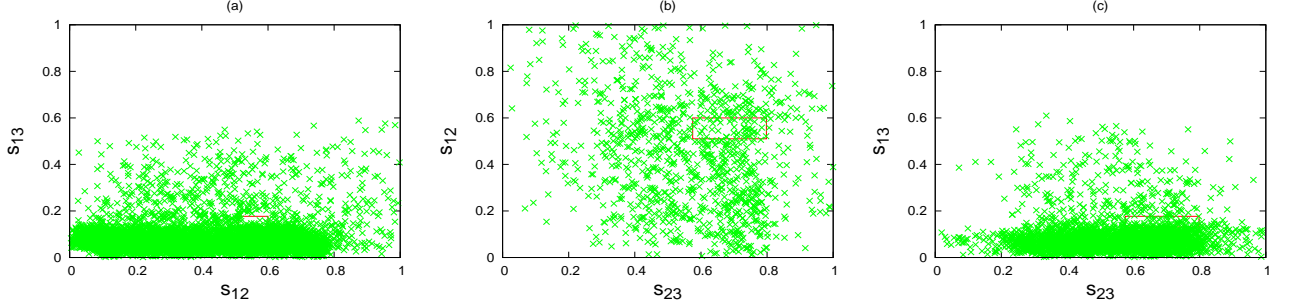


Figure 2: Plots showing the parameter space for any two mixing angles for texture two zero mass matrices (normal hierarchy).

one is constrained by its 1σ experimental bound for normal neutrino mass hierarchy. The rectangular regions in these plots show the experimentally allowed 3σ regions of the plotted angles. A general look at the figure (2) reveals that the structure (1) is compatible with the normal neutrino mass hierarchy.

Further, in figures 3(a) and 3(b) we present the plots showing the variation of the mixing angle s_{13} with the parameters C_l/m_e and C_ν/m_1 respectively for structure given in eqn.(1) pertaining to normal hierarchy of neutrino masses. While plotting these figures, the other two mixing angles have been constrained by their 3σ experimental bounds, while all the free parameters have been given full variation. The two parallel lines in these figures show the 3σ allowed range for the mixing angle s_{13} . Taking a careful look at these plots, one can note that the leptonic mixing angles donot have much dependence on the parameters C_l and C_ν . Further one can see that a fit for all the three mixing angles can be obtained for the values of parameters being $C_l \lesssim 0.7m_e$ and $C_\nu \lesssim 0.8m_1$.

Therefore, one can conclude that very small values of the parameters C_l and C_ν are required to fit the latest experimental data for the mass matrices pertaining to structure (1) and these essentially reduce to

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & D_l & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & D_\nu & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}. \quad (8)$$

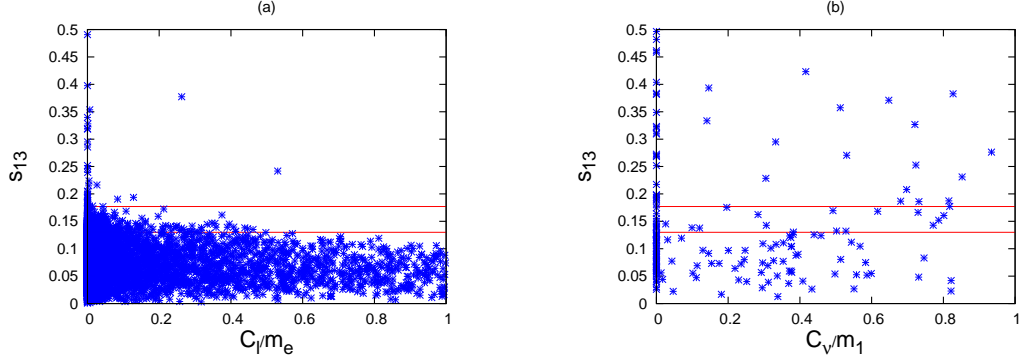


Figure 3: Plots showing the variation of the mixing angle s_{13} with the (1,1) element of the charged lepton mass matrix and the neutrino mass matrix for texture two zero mass matrices (normal hierarchy).

The structure (8) is referred to as Fritzsch-like texture four zero structure and is studied extensively in literature [25]. However, no such attempt has been made after the recent measurement of the mixing angle s_{13} . Therefore, it becomes interesting to analyse this structure in detail for its compatibility with the latest lepton mixing data.

4.2 Texture four zero lepton mass matrices

As discussed in the previous section, starting with the most general lepton mass matrices having ‘natural hierarchy’ [26] among their elements, one essentially arrives at the Fritzsch-like texture four zero structure as given in eqn.(8). In this context, it is interesting to note that using the facility of WB transformation performed by permutation matrices isomorphic to S_3 , all possible texture four zero mass matrices can essentially be classified into four distinct classes [22]. All the matrices belonging to a particular class have the same physical content. From the table (2) presented in [23] it is clear that class IV is not viable as all the matrices in this class correspond to the scenario where one of the generations gets decoupled from the other two. Therefore, in the following subsection we attempt to confront classes I, II, III of lepton mass matrices with the latest experimental data.

4.2.1 Class I ansatz

To begin with, we carry out a detailed analysis for texture four zero mass matrices belonging to class I, i.e.,

$$M_i = \begin{pmatrix} 0 & A_i e^{i\alpha_i} & 0 \\ A_i e^{-i\alpha_i} & D_i & B_i e^{i\beta_i} \\ 0 & B_i e^{-i\beta_i} & E_i \end{pmatrix}, \quad (9)$$

where $i = l, \nu_D$ corresponds to the charged lepton and Dirac neutrino mass matrices respectively. For the purpose of calculations, the elements D_l, D_ν as well as the phases ϕ_1 and ϕ_2 have been considered as free parameters. Following the methodology as discussed in section (2), we attempt to carry out a detailed study pertaining to normal, inverted as well as degenerate neutrino mass orderings. Firstly, we examine the compatibility of mass matrices given in eqn.(8) with the inverted hierarchy of neutrino masses. For this purpose in figure (4), we present the parameter space corresponding to the mixing angles s_{12} and s_{13} while s_{23} is being constrained by its 3σ range. The blank rectangular regions in these figures represent the 3σ allowed ranges of the mixing angles s_{12} and s_{13} . As can be seen from this plot, the plotted parameter space does not show any overlap with the experimentally allowed region. Therefore, we find that the class I ansatz of texture four zero lepton mass matrices is ruled out for the inverted hierarchy scenario of neutrino masses.

After ruling out the structure presented in eqn.(9) for the inverted hierarchy, we now proceed to examine the compatibility of these matrices for the normal hierarchy case. To this end, in figure (5) we present the plots showing the parameter space corresponding to any two mixing angles wherein the third one is constrained by its 1σ range. Interestingly, normal hierarchy seems to be viable in this case as can be seen from these plots, wherein the parameter space shows significant overlap with the experimentally allowed 3σ region shown by the rectangular boxes in each plot.

Further, in figure (6) we present the graphs showing the variation of the lightest neutrino mass and the effective Majorana mass with the mixing angle s_{13} for normal

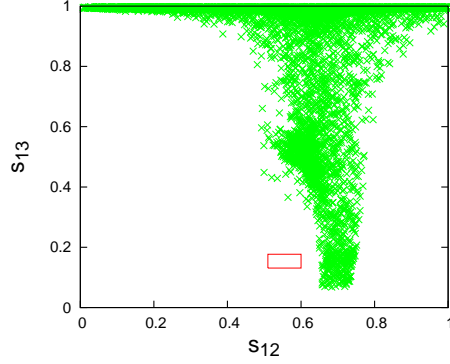


Figure 4: Plot showing the parameter space for s_{12} and s_{13} when s_{23} is constrained by its 3σ range for Class I ansatz of texture four zero mass matrices (inverted hierarchy).

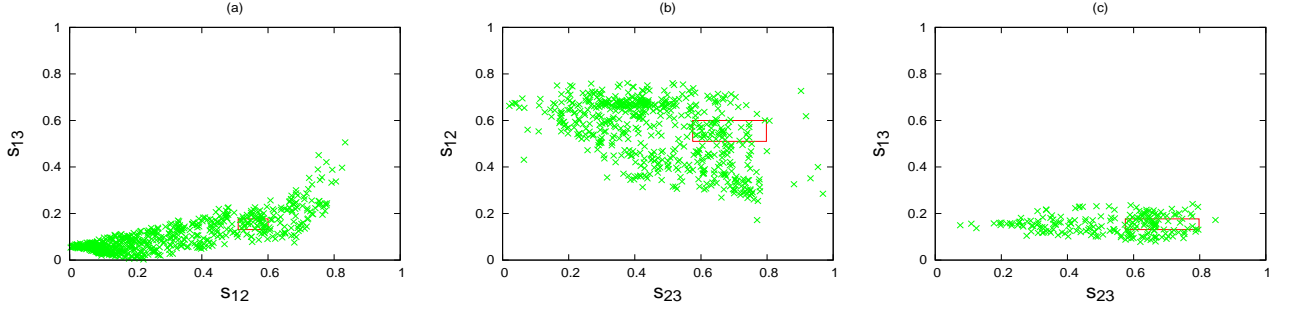


Figure 5: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class I ansatz of texture four zero mass matrices (normal hierarchy).

hierarchy, keeping the other two mixing angles constrained by their 3σ bounds. The parallel lines in each plot show the 3σ range of the mixing angle s_{13} . Taking a careful look at figure 6(a), one can find upper and lower bounds on the lightest neutrino mass to be $0.01eV \lesssim m_{\nu 1} \lesssim 0.1eV$ approximately. Similarly, from the numerical results shown in figure 6(b), one can obtain an approximate lower bound on the effective Majorana mass $|m_{ee}|$, viz. $|m_{ee}| \gtrsim 10^{-5}eV$.

The degenerate scenario of the neutrino masses can be characterized by either $m_{\nu 1} \lesssim m_{\nu 2} \sim m_{\nu 3} \sim 0.1eV$ or $m_{\nu 3} \sim m_{\nu 1} \lesssim m_{\nu 2} \sim 0.1eV$, corresponding to normal hierarchy and inverted hierarchy respectively. Since while carrying out the calculations pertaining to both the normal as well as inverted hierarchy cases, the range of

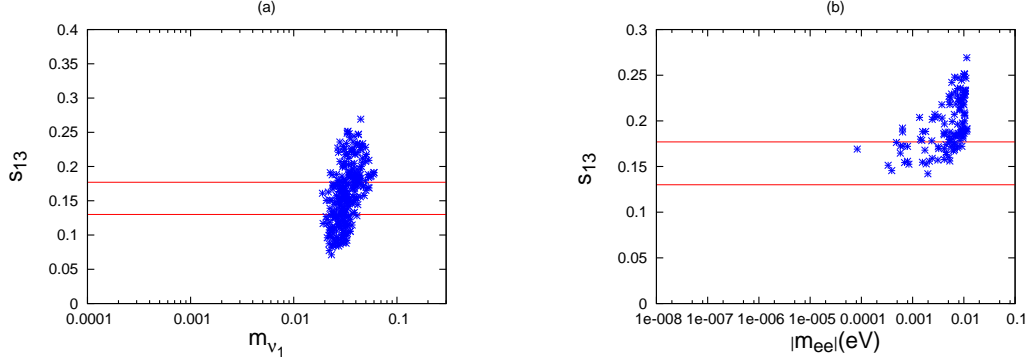


Figure 6: Plots showing the dependence of mixing angle s_{13} on (a) the lightest neutrino mass and (b) the effective Majorana mass, when the other two angles are constrained by their 3σ ranges for Class I ansatz of texture four zero mass matrices (normal hierarchy).

lightest neutrino mass is taken to be $10^{-8} - 10^{-1}$ eV, which includes the neutrino masses corresponding to the degenerate scenario; therefore, by discussion similar to the one given for ruling out inverted hierarchy, degenerate scenario corresponding to inverted hierarchy of neutrino masses seems to be ruled out for class I ansatz of texture four zero mass matrices. However, from figure (6) the value of the lightest neutrino mass pertaining to the degenerate scenario, $m_{\nu 1} \sim 0.1 \text{ eV}$ seems to be included in the experimentally allowed region, thereby showing the viability of Class I ansatz for degenerate scenario pertaining to normal hierarchy.

4.2.2 Class II ansatz

To analyse this class we follow the same procedure as for class I Ansätze. The charged lepton and neutrino mass matrices which we choose to analyse for this class can be given as,

$$M_i = \begin{pmatrix} D_i & A_i e^{i\alpha_i} & 0 \\ A_i e^{-i\alpha_i} & 0 & B_i e^{i\beta_i} \\ 0 & B_i e^{-i\beta_i} & E_i \end{pmatrix}, \quad (10)$$

where $i = l, \nu_D$ corresponds to the charged lepton and Dirac neutrino mass matrices respectively. As for class I, the elements D_l and D_ν as well as the phases ϕ_1 and ϕ_2 are considered to be the free parameters. Firstly, we examine the viability of

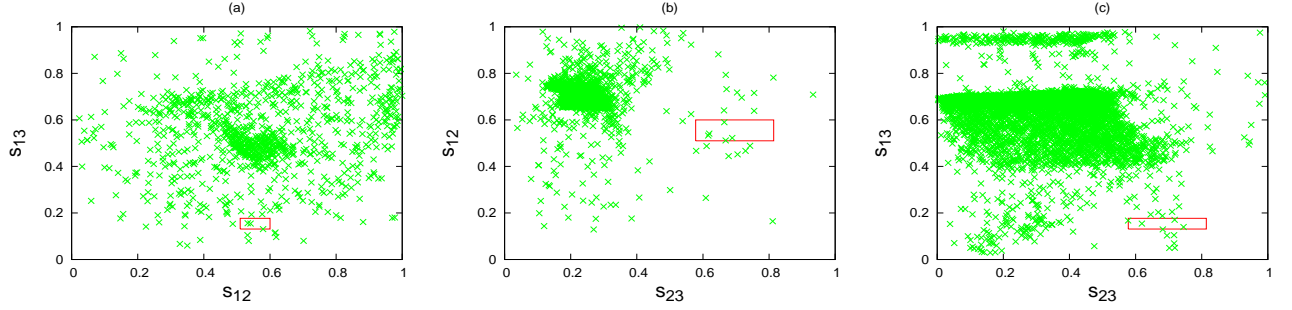


Figure 7: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class II ansatz of texture four zero mass matrices (inverted hierarchy).

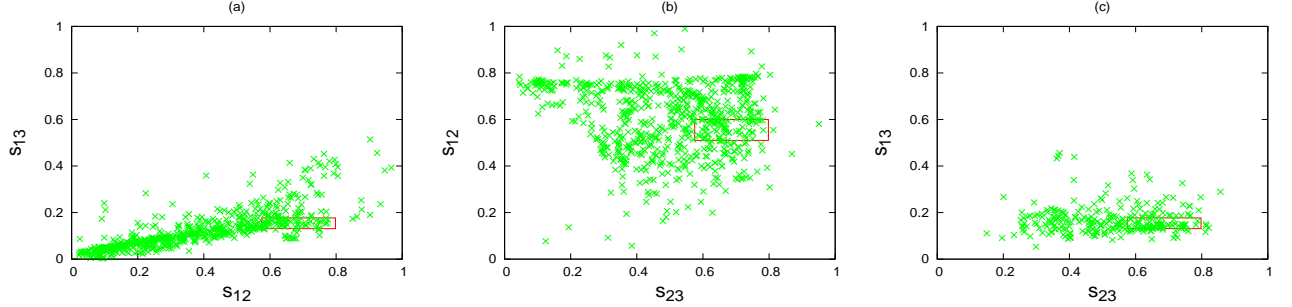


Figure 8: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class II ansatz of texture four zero mass matrices (normal hierarchy).

inverted hierarchy for the mass matrices presented in eqn.(10). To this end in figure (7), we present the plots showing the parameter space for two mixing angles wherein the third angle is constrained by its 1σ range. The rectangular boxes in these plots show the 3σ ranges for the two mixing angles being considered. It is interesting to note that the inverted hierarchy seems to be compatible with the 1σ ranges of the present lepton mixing data.

Next, in order to examine the compatibility of structure given in eqn.(10) with the normal hierarchy, in figure (8) we present the plots showing the parameter space for two mixing angles wherein the third angle is constrained by its 1σ experimental range. A general look at figure (8) reveals that the mass matrices presented in eqn.(10) are compatible with the normal hierarchy scenario of neutrino masses.

Thus, we find that for texture four zero matrices pertaining to class II, both

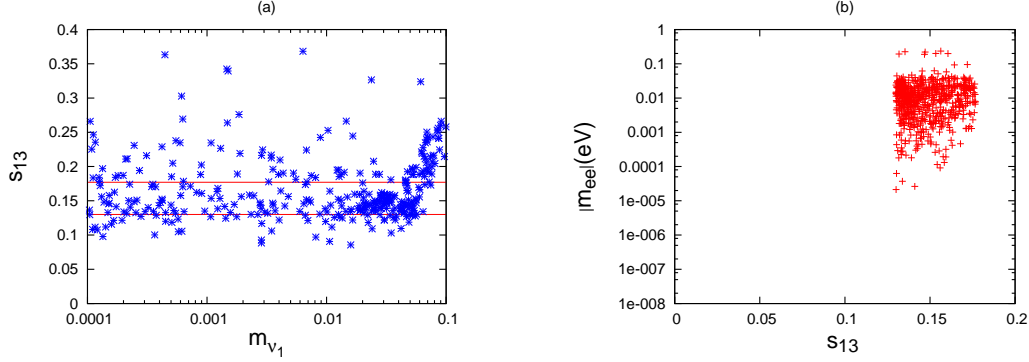


Figure 9: Plots showing the dependence of mixing angle s_{13} on (a) the lightest neutrino mass and (b) the effective Majorana mass, for Class II ansatz of texture four zero mass matrices (normal hierarchy).

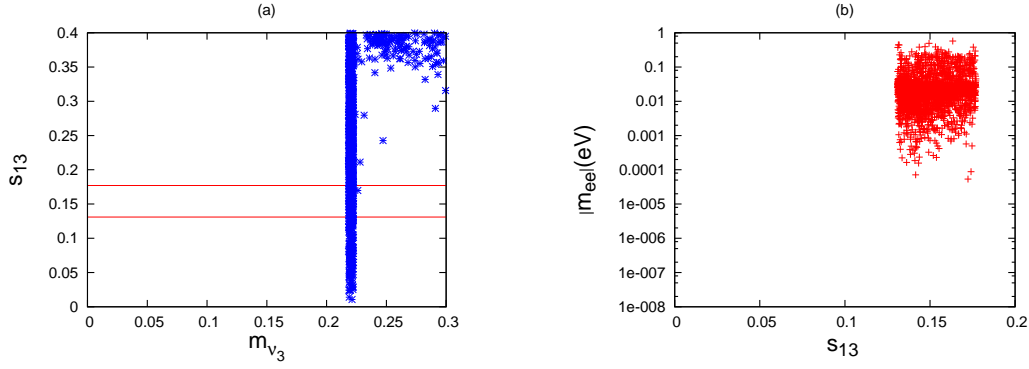


Figure 10: Plots showing the dependence of mixing angle s_{13} on (a) the lightest neutrino mass and (b) the effective Majorana mass, for Class II ansatz of texture four zero mass matrices (inverted hierarchy).

the normal as well as inverted neutrino mass hierarchies are compatible the lepton mixing data. As a next step, in figure (9) we attempt to study the implications of the 3σ ranges of the leptonic mixing angle s_{13} on the lightest neutrino mass and the effective Majorana mass respectively for normal hierarchy scenario of neutrino masses. Analogous plots for inverted hierarchy scenario have been presented in figure (10). While plotting figures 9(a) and 10(a), the other two mixing angles have been constrained by their 3σ experimental bounds, whereas figures 9(b) and 10(b) have been obtained by constraining s_{13} by its 3σ experimental bound. Interestingly, one finds that the present 3σ ranges of the mixing angles provide no bound on the lightest neutrino mass for the normal hierarchy respectively, whereas for the inverted

hierarchy case one gets an extremely narrow range for the lightest neutrino mass which clearly points to the degenerate scenario of neutrino masses. Interestingly, a careful look at figures 9(b) and 10(b) reveals that the 3σ range for s_{13} provides an approximate lower bound on the effective Majorana mass $|m_{ee}|$, viz. $|m_{ee}| \gtrsim 10^{-5}eV$ and $|m_{ee}| \gtrsim 10^{-4}eV$ for the normal and inverted hierarchy scenarios respectively.

4.2.3 Class III ansatz

To study the lepton mass matrices for this class, we choose to analyse the following structure,

$$M_i = \begin{pmatrix} 0 & A_i e^{i\alpha_i} & B_i e^{i\gamma_i} \\ A_i e^{-i\alpha_i} & 0 & D_i e^{i\beta_i} \\ B_i e^{-i\gamma_i} & D_i e^{-i\beta_i} & E_i \end{pmatrix}, \quad (11)$$

where $i = l, \nu_D$ corresponds to the charged lepton and Dirac neutrino mass matrices respectively. In the case of factorizable phases, the lepton mass matrices belonging to this class can be analysed following a methodology similar to that for class I and class II ansatz. For the purpose of calculations, the (2,3) element in each sector, D_l and D_ν , as well as the phases ϕ_1 and ϕ_2 have been considered as free parameters. Firstly, we examine the texture four zero mass matrices in this class for their compatibility with the inverted hierarchy scenario. For this purpose, in figure (11) we present the plots showing the parameter space for any two mixing angles keeping the third one constrained by its 1σ experimental bound. A general look at figure (11) shows that the inverted mass neutrino mass ordering scenario is viable as can be seen from significant overlap with the experimentally allowed 3σ regions shown by rectangular boxes in each plot.

After examining the viability of inverted hierarchy scenario for the texture four zero mass matrices pertaining to class III, we now proceed to examine their compatibility with the normal hierarchy of neutrino masses. To this end, in figure (12) we present the plot showing the parameter space corresponding to the mixing angles s_{23} and s_{12} . While plotting this figure, the mixing angle s_{13} has been constrained

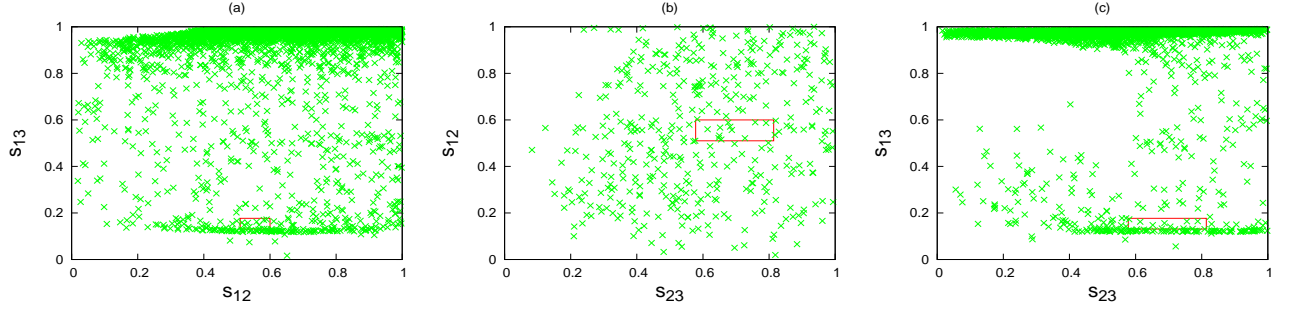


Figure 11: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range for Class III ansatz of texture four zero mass matrices (inverted hierarchy).

by its 1σ experimental bound. A general look at figure (12) reveals that normal hierarchy is ruled out by the 1σ ranges of the latest lepton mixing data. This can be understood by noting that the plotted parameter space of the two angles has no overlap with their experimentally allowed 1σ region shown by the rectangular box in the figure.

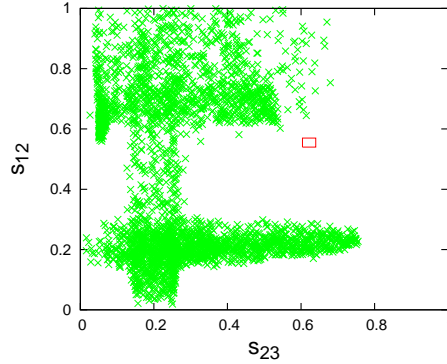


Figure 12: Plot showing the parameter space for s_{23} and s_{12} when s_{13} is constrained by its 1σ range for Class III ansatz of texture four zero mass matrices (normal hierarchy).

Further, we examine the compatibility of these mass matrices with the normal hierarchy scenario for the 3σ ranges of the present lepton mixing data. For this purpose in figure (13), we present the parameter space corresponding to any two mixing angles while the third one being constrained by its 3σ range. The blank rectangular regions in figure (13) represent the 3σ allowed ranges of the two mixing

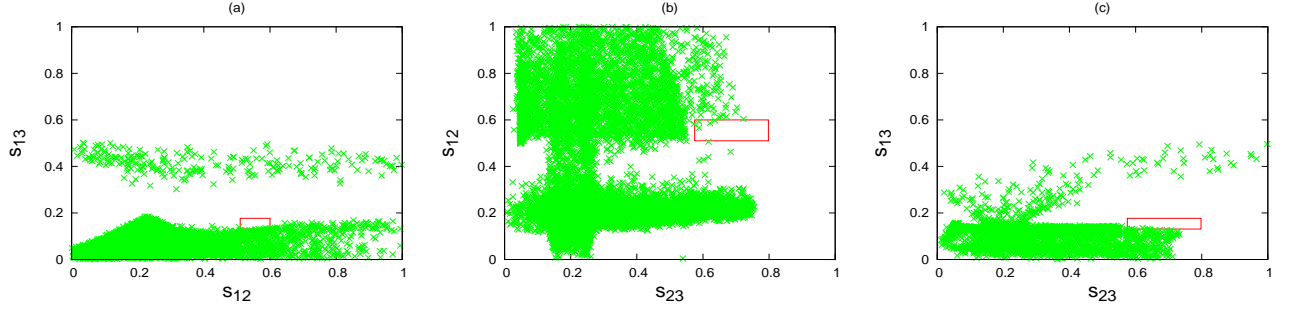


Figure 13: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range for Class III ansatz of texture four zero mass matrices (normal hierarchy).

angles being considered respectively. Interestingly, we find that the class III ansatz is compatible with the 3σ ranges of the present lepton mixing data pertaining to normal hierarchy scenario.

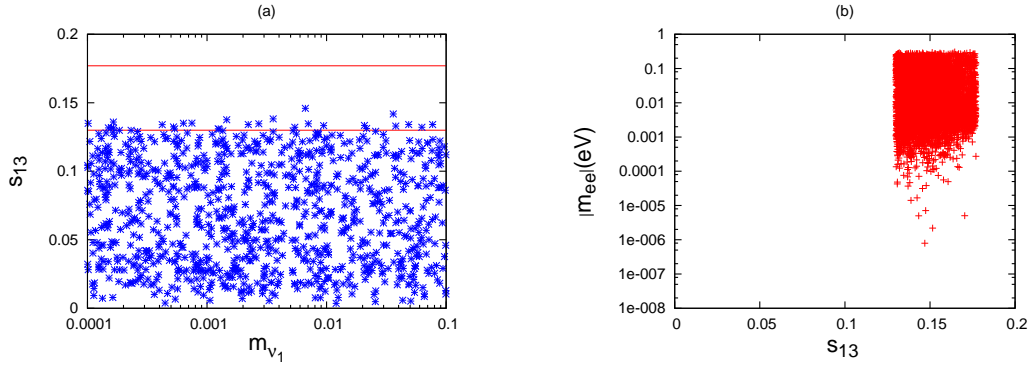


Figure 14: Plots showing the dependence of mixing angle s_{13} on (a) the lightest neutrino mass and (b) the effective Majorana mass, for Class III ansatz of texture four zero mass matrices (normal hierarchy).

As a next step, in figure 14(a) and 14(b) we attempt to study the implications of the 3σ ranges of the leptonic mixing angle s_{13} on the lightest neutrino mass and the effective Majorana mass respectively for normal hierarchy scenario of neutrino masses. Analogous plots for inverted hierarchy scenario have been presented in figure (15). While plotting figures 14(a) and 15(a), the other two mixing angles have been constrained by their 3σ experimental bounds, whereas figures 14(b) and 15(b) have been obtained by constraining s_{13} by its 3σ experimental bound. Interestingly, one finds that the present 3σ range of the mixing angle s_{13} provides no bound on the

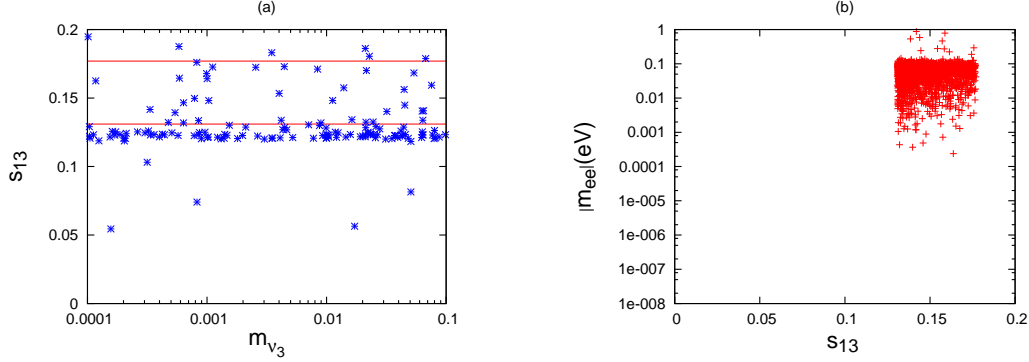


Figure 15: Plots showing the dependence of mixing angle s_{13} on (a) the lightest neutrino mass and (b) the effective Majorana mass, for Class III ansatz of texture four zero mass matrices (inverted hierarchy).

lightest neutrino mass for both the normal as well as inverted hierarchy scenario. Further, a careful look at figures 14(b) and 15(b) reveals the lower bound on $|m_{ee}|$ for the case of normal hierarchy is quite broader as compared to that for the inverted hierarchy scenario, viz., $|m_{ee}| \gtrsim 10^{-6} eV$ and $|m_{ee}| \gtrsim 10^{-4} eV$ for normal and inverted hierarchy respectively. Further, since the lightest neutrino mass for both normal as well as inverted hierarchy is unrestricted, therefore possibility of degenerate neutrino mass ordering can not be ruled out for class III ansatz of texture four zero mass matrices.

4.3 Texture five zero lepton mass matrices

After studying all possible texture four zero lepton mass matrices, it becomes interesting to explore the parallel texture five zero structures for each class which can be derived by substituting either $D_l = 0$, $D_\nu \neq 0$ or $D_l \neq 0$, $D_\nu = 0$ in the corresponding texture four zero mass matrices. In this subsection, we carry out a detailed study pertaining to all classes of texture five zero lepton mass matrices for both the possibilities leading to texture five zero structures. A detailed analysis [21] for both the cases of Fritzsch-like texture five zero mass matrices has recently been carried out. Therefore, in the present work we present the analyses for class II and class III ansatz of texture five zero mass matrices only.

4.3.1 Class II ansatz

The two possibilities for texture five zero lepton mass matrices for this class can be given as,

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} D_\nu & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}, \quad (12)$$

or

$$M_l = \begin{pmatrix} D_l & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & E_\nu \end{pmatrix}, \quad (13)$$

We study both these possibilities in detail for all the neutrino mass orderings. Firstly, we examine the compatibility of matrices given in equations (12) and (13) with the inverted hierarchy of neutrino masses. For this purpose, in figures 16(a) and 16(b), we present the plots showing the parameter space allowed by this ansatz for the mixing angles s_{12} and s_{13} for the cases $D_l = 0$, $D_\nu \neq 0$ and $D_l \neq 0$, $D_\nu = 0$ respectively. While plotting these graphs, the mixing angle s_{23} has been constrained by its 3σ experimental bound for inverted hierarchy of neutrino masses. The rectangular regions in these plots represent the 3σ experimental ranges for s_{12} and s_{13} . A general look at these plots reveals that inverted hierarchy seems to be viable for for the $D_l \neq 0$ and $D_\nu = 0$ case, however it is clearly ruled out for the other possibilty i.e. $D_l = 0$ and $D_\nu \neq 0$.

After examining the viability of inverted hierarchy for both the cases of texture five zero mass matrices pertaining to class II, we now proceed to examine their compatibility with the normal hierarchy scenario. To this end, in figures (17) and (18), we present the plots showing the parameter space allowed by this ansatz for any two mixing angles wherein the third one is constrained by its 1σ experimental bound for normal hierarchy of neutrino masses. The rectangular regions in these plots represent the 3σ ranges for the two mixing angles being considered. Normal hierarchy seems to be viable for both the cases as can be seen from the significant overlap with the experimentally allowed region.

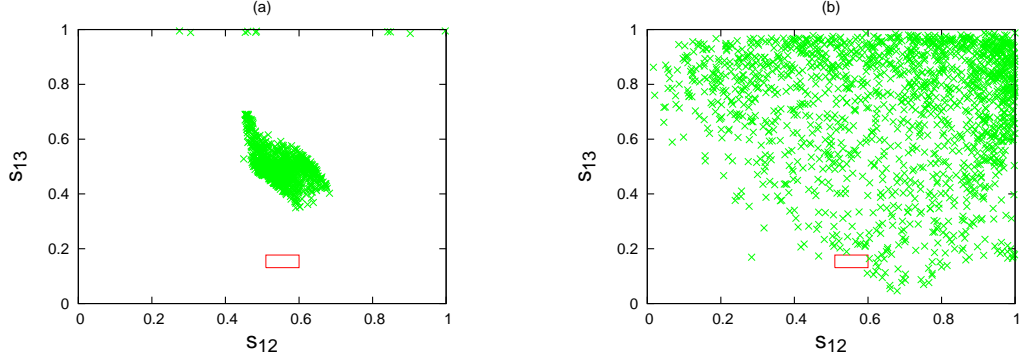


Figure 16: Plots showing the parameter space for the mixing angles s_{12} and s_{13} when s_{23} is constrained by its 3σ range for Class II ansatz of texture five zero mass matrices pertaining to the possibility (a) $D_l = 0$ and $D_\nu \neq 0$ (b) $D_l \neq 0$ and $D_\nu = 0$ (inverted hierarchy).

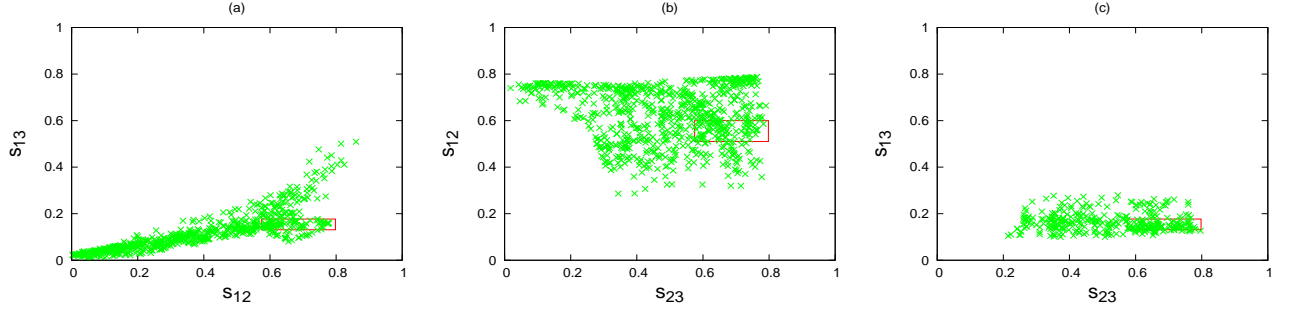


Figure 17: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class II ansatz of texture five zero mass matrices (normal hierarchy).

Further, in figure (19) we present the graphs showing the variation of the lightest neutrino mass with the mixing angle s_{13} for both the cases $D_l = 0$ and $D_\nu \neq 0$ as well as $D_l \neq 0$ and $D_\nu = 0$ pertaining to normal hierarchy of neutrino masses. While plotting these graphs, the other two mixing angles have been constrained by their 3σ experimental bounds. The parallel lines in each plot show the 3σ range of the mixing angle s_{13} . Interestingly, taking a careful look at these graphs one finds that for the $D_l = 0$ and $D_\nu \neq 0$ case the lightest neutrino mass is largely unrestricted, whereas for the $D_l \neq 0$ and $D_\nu = 0$ case a lower bound $\approx 0.005eV$ can be obtained. As a next step, in figures 20(a) and 20(b) we study the variation of the effective Majorana mass with the s_{13} for the cases $D_l = 0$, $D_\nu \neq 0$ and $D_l \neq 0$ and $D_\nu = 0$ respectively

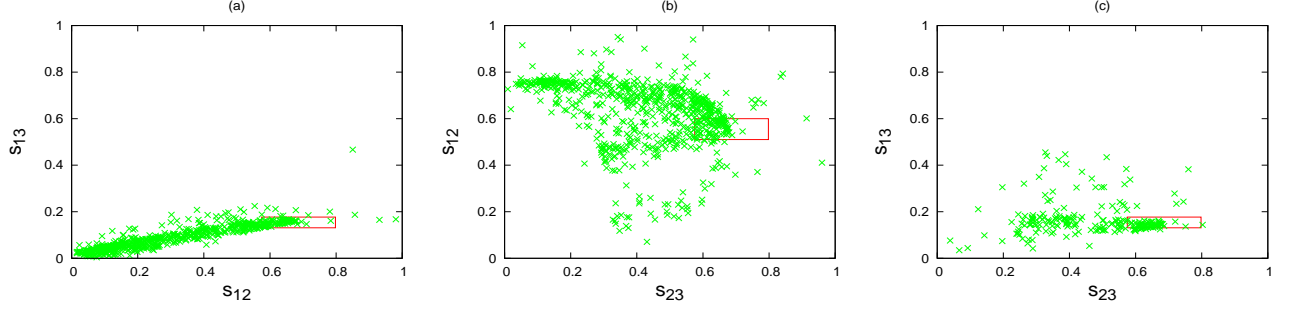


Figure 18: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 1σ range in $D_l \neq 0$ and $D_\nu = 0$ scenario for Class II ansatz of texture five zero mass matrices (normal hierarchy).

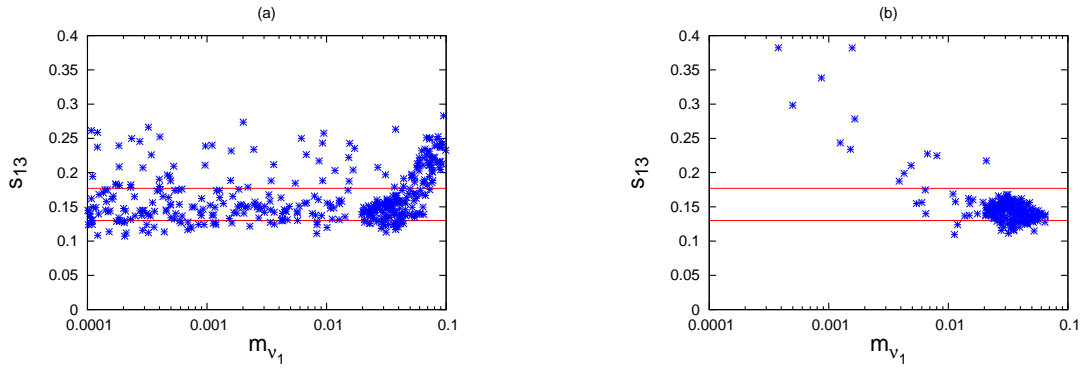


Figure 19: Plots showing the dependence of mixing angle s_{13} on the lightest neutrino mass when the other two angles are constrained by their 3σ ranges for Class II ansatz of texture five zero mass matrices pertaining to the possibility (a) $D_l = 0$ and $D_\nu \neq 0$ (b) $D_l \neq 0$ and $D_\nu = 0$ (normal hierarchy).

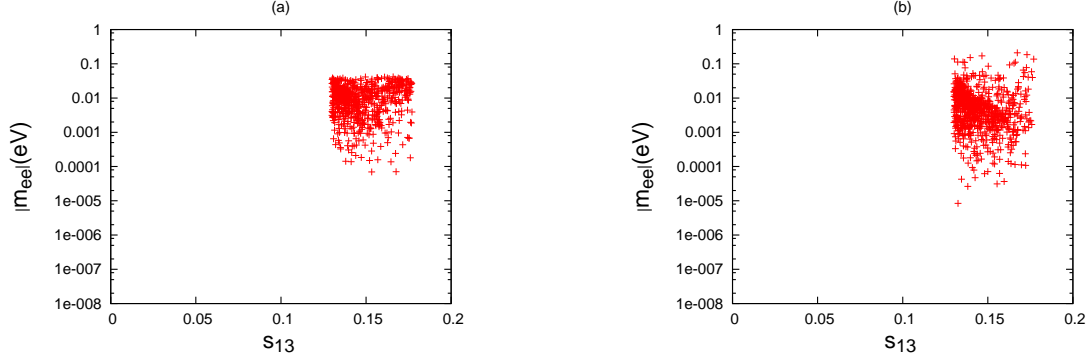


Figure 20: Plots showing the variation of the effective Majorana mass measured in neutrinoless double beta decay with s_{13} for Class II ansatz of texture five zero mass matrices pertaining to the possibility (a) $D_l = 0$ and $D_\nu \neq 0$ (b) $D_l \neq 0$ and $D_\nu = 0$ (normal hierarchy)

texture five zero mass matrices in class II pertaining to normal hierarchy of neutrino masses. While plotting these figures, the mixing angle s_{13} has been constrained by its 3σ experimental bound. A careful look at these figures reveals that the 3σ range for s_{13} provides a lower bound $\approx 10^{-4}eV$ and $\approx 10^{-5}eV$ for $|m_{ee}|$ pertaining to the $D_l = 0, D_\nu \neq 0$ and $D_l \neq 0, D_\nu = 0$ cases respectively.

4.3.2 Class III ansatz

The two possibilities for texture five zero lepton mass matrices for this class can be given as,

$$M_l = \begin{pmatrix} 0 & A_l e^{i\alpha_l} & B_l \\ A_l e^{-i\alpha_l} & 0 & 0 \\ B_l & 0 & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & A_\nu e^{i\alpha_\nu} & B_\nu \\ A_\nu e^{-i\alpha_\nu} & 0 & D_\nu e^{i\beta_\nu} \\ B_\nu & D_\nu e^{-i\beta_\nu} & E_\nu \end{pmatrix}, \quad (14)$$

or

$$M_l = \begin{pmatrix} 0 & A_l e^{i\alpha_l} & B_l \\ A_l e^{-i\alpha_l} & 0 & D_l e^{i\beta_l} \\ B_l & D_l e^{-i\beta_l} & E_l \end{pmatrix}, \quad M_\nu = \begin{pmatrix} 0 & A_\nu e^{i\alpha_\nu} & B_\nu \\ A_\nu e^{-i\alpha_\nu} & 0 & 0 \\ B_\nu & 0 & E_\nu \end{pmatrix}, \quad (15)$$

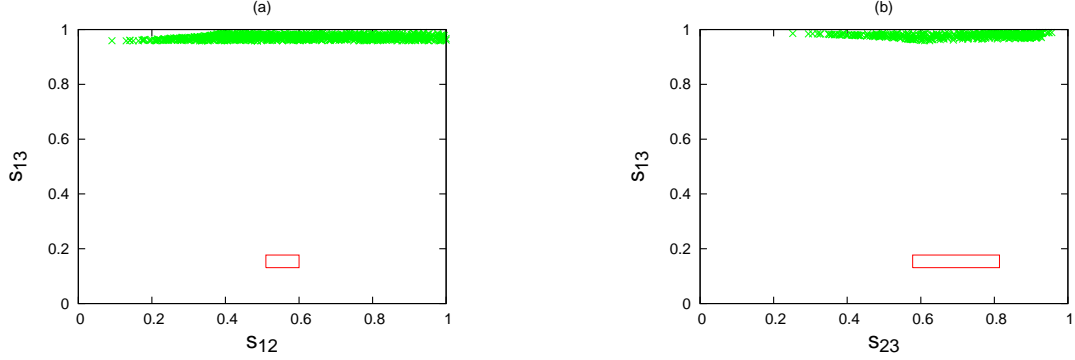


Figure 21: Plots showing the parameter space for (a) s_{12} and s_{13} (b) s_{23} and s_{13} in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class III ansatz of texture five zero mass matrices (inverted hierarchy).

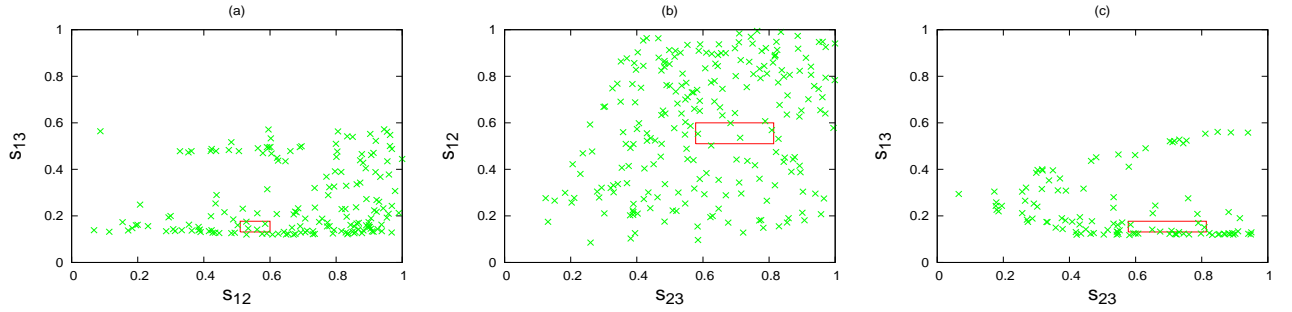


Figure 22: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range in the $D_l \neq 0$ and $D_\nu = 0$ scenario for Class III ansatz of texture five zero mass matrices (inverted hierarchy).

We study both these possibilities in detail for all the neutrino mass orderings. Firstly, we examine the compatibility of matrices given in equations (14) and (15) with the inverted hierarchy of neutrino masses. For this purpose, in figures (21) and (22), we present the plots showing the parameter space allowed by this ansatz, corresponding to the cases $D_l = 0$, $D_\nu \neq 0$ and $D_l \neq 0$, $D_\nu = 0$ respectively, for any two mixing angles wherein the third one is constrained by its 3σ experimental bound for inverted hierarchy of neutrino masses. The rectangular regions in these plots represent the 3σ ranges for the two mixing angles being considered. Interestingly, one finds that for the case $D_l = 0$ and $D_\nu \neq 0$ of texture five zero lepton mass matrices inverted hierarchy is ruled out, whereas for the case $D_l \neq 0$ and $D_\nu = 0$ of texture five zero lepton mass matrices inverted hierarchy scenario seems to be viable.

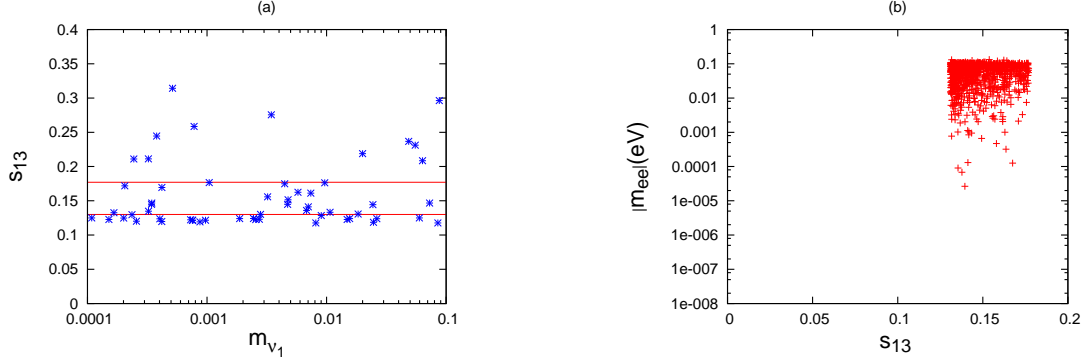


Figure 23: Plots showing the dependence of (a) the lightest neutrino mass (b) effective Majorana mass on the mixing angle s_{13} in the $D_l \neq 0$ and $D_\nu = 0$ scenario for Class III ansatz of texture five zero mass matrices (inverted hierarchy).

For the $D_l \neq 0$ and $D_\nu = 0$ case of lepton mass matrices, wherein inverted hierarchy is shown to be viable, in figures 23(a) and 23(b) we present the plots showing the dependence of the lightest neutrino mass and the effective Majorana mass respectively on the leptonic mixing angle s_{13} . Interestingly, one finds that the lightest neutrino mass is unrestricted for this structure, whereas a lower bound $\approx 10^{-4}eV$ can be obtained for the effective Majorana mass $|m_{ee}|$.

After studying both the cases for texture five zero mass matrices for inverted hierarchy pertaining to class III ansatz, we now carry out a similar analysis pertaining to normal hierarchy. To this end, in figures (24) and (25), we present the plots showing the parameter space corresponding to any two mixing angles wherein the third one is constrained by its 3σ range. Interestingly, normal hierarchy seems to be ruled out for the case $D_l \neq 0$ and $D_\nu = 0$, whereas for the case $D_l = 0$ and $D_\nu \neq 0$ of texture five zero lepton mass matrices normal hierarchy seems to be viable.

For the $D_l = 0$ and $D_\nu \neq 0$ case of lepton mass matrices, wherein normal hierarchy is shown to be viable, in figures 26(a) and 26(b) we present the plots showing the dependence of the lightest neutrino mass and the effective Majorana mass respectively on the leptonic mixing angle s_{13} . Interestingly, one finds that the lightest neutrino mass is unrestricted for this structure, whereas a lower bound $\approx 10^{-6}eV$ can be obtained for the effective Majorana mass $|m_{ee}|$.

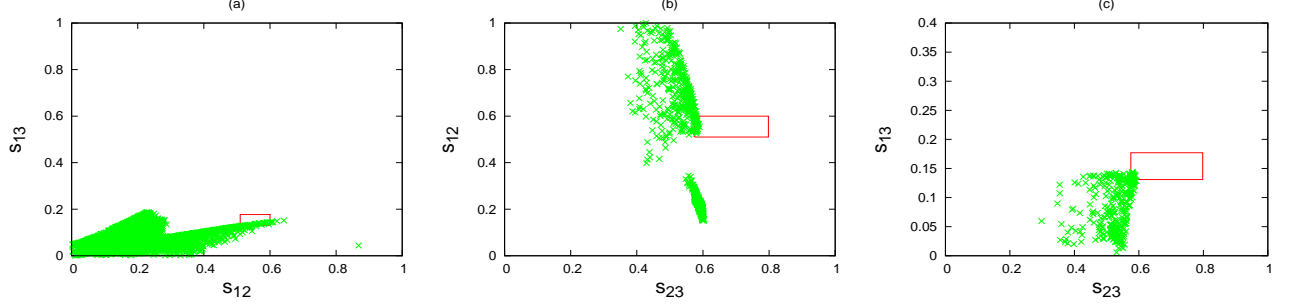


Figure 24: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range for normal hierarchy scenario in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class III ansatz of texture five zero mass matrices.

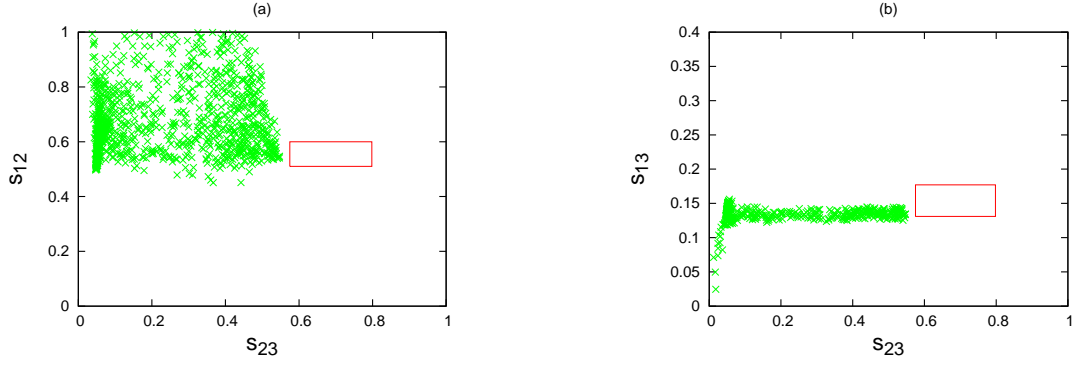


Figure 25: Plots showing the parameter space for any two mixing angles when the third angle is constrained by its 3σ range for normal hierarchy scenario in the $D_l \neq 0$ and $D_\nu = 0$ scenario for Class III ansatz of texture five zero mass matrices.

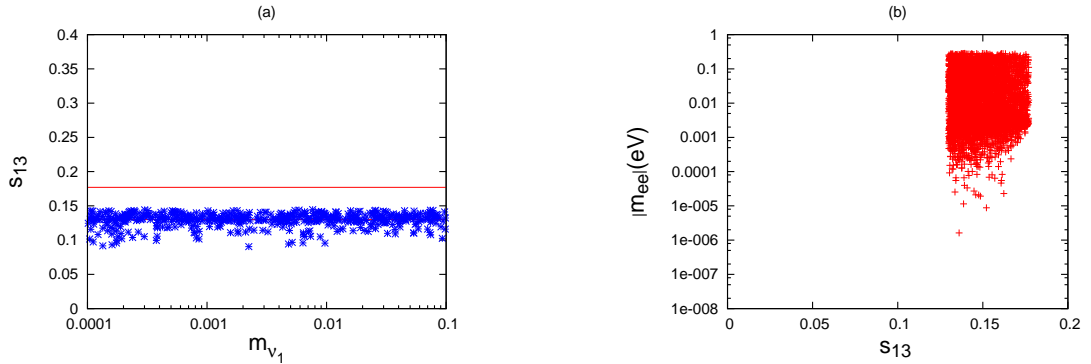


Figure 26: Plots showing the dependence of (a) the lightest neutrino mass (b) effective Majorana mass on the mixing angle s_{13} in the $D_l = 0$ and $D_\nu \neq 0$ scenario for Class III ansatz of texture five zero mass matrices (normal hierarchy) .

5 Summary and conclusions

To summarize, for Majorana neutrinos we have carried out detailed calculations pertaining to non minimal textures characterized by texture two zero Fritzsch-like structure as well as all possibilities for texture four zero and five zero lepton mass matrices. Corresponding to these, we have considered all the three possibilities for neutrino masses i.e. normal, inverted as well as degenerate scenarios. The compatibility of these texture specific mass matrices has been examined by plotting the parameter space corresponding to any two of the leptonic mixing angles. Further, for all the structures which seem to be compatible with the recent lepton mixing data, the implications of the mixing angles on the lightest neutrino mass as well as the effective Majorana mass measured in neutrinoless double beta decay have also been studied.

The analysis reveals that the Fritzsch like texture two zero lepton mass matrices are compatible with the recent lepton mixing data pertaining to normal as well as inverted hierarchy of neutrino masses. Interestingly, one finds that both the normal as well as inverted neutrino mass hierarchies are compatible with texture four zero mass matrices pertaining to class II and III contrary to the case for texture four zero mass matrices pertaining to class I wherein inverted hierarchy seems to be ruled out. Degenerate scenario pertaining to inverted hierarchy is clearly ruled out for texture four zero mass matrices pertaining to class I, whereas for normal hierarchy this scenario can not be ruled out. For texture four zero mass matrices in class II and III, none of the possibilities for degenerate neutrino mass scenario can be ruled out. Mass matrices in class IV are phenomenologically excluded.

For texture five zero lepton mass matrices, we analyse both the cases, viz. $D_l = 0, D_\nu \neq 0$ as well as $D_l \neq 0, D_\nu = 0$ for class II and class III ansatz. A detailed analysis for texture five zero matrices pertaining to class I has already been carried out in [21]. For class II, normal hierarchy is viable for both the cases while the inverted hierarchy seems to be ruled out for the case, $D_l = 0, D_\nu \neq 0$. Finally, for texture five zero mass matrices pertaining to class III we find that inverted hierarchy

is viable for the case $D_l \neq 0, D_\nu = 0$, while the normal hierarchy is compatible with the $D_l = 0, D_\nu \neq 0$ case. Refinements in the measurements of the lightest neutrino mass and $\langle m_{ee} \rangle$ are expected to have important implications for the texture specific mass matrices considered in the present work.

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